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CORRELATION OF THERMOPHORETICALLY-MODIFIED SMALL PARTICLE DIFFUSIONAL DEPOSITION RATES IN FORCED CONVECTION SYSTEMS WITH VARIABLE PROPERTIES, TRANSPIRATION COOLING AND/OR VISCOUS DISSIPATION

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Abstract—Rational correlations are developed and presented for making engineering predictions of small particle diffusional deposition rates in the presence of thermophoresis, transpiration cooling, viscous dissipation and/or variable fluid properties. We propose and find that the effects of these phenomena on the convective-diffusion mass transfer Stanton number can be simply expressed in the product form:

$$\frac{St_{\mathrm{m}}}{St_{\mathrm{m},0}} \cong fct_1\left\{\begin{matrix}\text{real and/or thermophoretic}\\ \text{`blowing' (suction)}\end{matrix}\right\}\cdot fct_2\left\{\begin{matrix}\text{`thermophoretic}\\ \text{sink' (source)}\end{matrix}\right\},$$

where the first function accounts for particle transport by convective processes normal to the deposition surface, and the second function accounts for the spatial variability of the thermophoretic drift velocity, as influenced by viscous dissipation (mainly outside of the Brownian diffusion sublayer). Correlations of this particular form, when tested against rigorous numerical boundary layer (BL) calculations, gave *average* errors of only 7% for all *laminar* BL cases ($0 \leqslant Ma_{\mathrm{e}} \leqslant 2$; $0.5 \leqslant T_{\mathrm{w}}/T_{\mathrm{e}} \leqslant 1$; $0 \leqslant \alpha_{\mathrm{T,p}}\,Le \leqslant 1.5$; $B \leqslant 5$), and 2% for all *turbulent* BL cases ($0.5 \leqslant T_{\mathrm{w}}/T_{\mathrm{e}} \leqslant 1$; $0 \leqslant \alpha_{\mathrm{T,p}}\,Le \leqslant 1.5$; $0.9 \times 10^4 \leqslant Re \leqslant 3 \times 10^7$; $B \leqslant 5$), respectively.

NOMENCLATURE

B	combined ('real' and 'thermophoretic') blowing (suction) parameter, equation (16)
B_{m}	'real' blowing (suction) parameter, equation (13)
$-B_{\mathrm{T}}$	'thermophoretic suction' parameter, equation (8)
D	Brownian (Fick) diffusion coefficient
Da	Damköhler number based on k_{eff}, equation (10)
Eu	Euler number, d ln u_{e}/d ln x
j''	diffusion mass flux, equations (1) and (2)
k_{eff}	pseudo-first order 'reaction' rate constant, equation (4)
Kn	particle Knudsen number (ratio of particle diameter to carrier gas mean-free-path)
Le	Lewis number (ratio of particle Brownian diffusivity to carrier gas heat diffusivity)
Ma	Mach number
p	pressure
Pr	Prandtl number (ratio of carrier gas momentum diffusivity to heat diffusivity)
r	recovery factor for mainstream kinetic energy, equation (8)
Re	Reynolds number
Sc	Schmidt number (ratio of gas momentum diffusivity to particle Brownian diffusivity)
St	Stanton number [for mass transfer, equations (1) and (2b), unless otherwise specified, e.g. St_{h}]
T	absolute temperature
u	fluid velocity in x-direction (parallel to wall), cf. Fig. 1
v_{T}	thermophoretic drift velocity, equation (3)
v_{w}	wall blowing velocity normal to surface
x	distance measured along body surface, Fig. 1
y	distance normal to surface, cf. Fig. 1.

Greek symbols

α	thermal (heat) diffusivity of gas
α_{T}	thermal diffusion factor of particle, cf. ref [1]
$\alpha_{\mathrm{T}}\,Le$	ratio of thermophoretic diffusivity to carrier gas heat diffusivity, cf. ref [21]
δ	boundary layer thickness, equation (8a)
γ	carrier gas specific heat ratio
ρ	density of gas
ω	mass fraction of particles in prevailing mixture.

Subscripts

aw	adiabatic wall condition
B	mass transfer through the surface (blowing or suction)

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e	outer edge of the BL (local 'mainstream'), cf. Fig. 1
h	heat transfer
m	outer edge of the Brownian diffusion (mass transfer) BL, or mass transfer
p	particle
T	thermophoresis
w	at the surface (wall)
0	conditions without thermophoresis, blowing (suction) and viscous dissipation (e.g. St_0)
1	blowing (suction) effect
2	'pseudo-sink' effect.

Superscripts

—	averaged quantity
s	exponent of Le in relationship among Brownian diffusion sublayer thickness, particle size and the 'strength' of thermophoresis, cf. equation (10).

Miscellaneous

BL	boundary layer (L, laminar; T, turbulent)
$fct\{\ \}$	function of argument indicated in $\{\ \}$.

1. INTRODUCTION

Vapor and/or particle *mass* transport (deposition) is central to a variety of engineering fields, including materials technology (electronic device fabrication, chemical coating of metals, etc.), gas 'clean-up' (e.g. filtration), and the corrosion and/or fouling of heat exchange or gas turbine equipment. In the latter cases, the sharply increasing cost of high-grade fuels has stimulated a renewed interest in the combustion of coal-derived or lower grade 'residual' petroleum-derived fuels, each associated with inorganic impurities which form ash or salt deposits. At the same time, the need for higher turbine inlet temperatures required for lower fuel consumption demands novel, more efficient turbine blade cooling schemes, such as film cooling or full-coverage transpiration. Under these combined circumstances thermal diffusion ('thermophoresis' for small particles) plays an important role in determining mass deposition rates, as demonstrated (principally in the absence of viscous dissipation and/or transpiration or film cooling) in refs. [1–3]. Viscous dissipation, known to modify temperature profiles within thermal boundary layers (BL) under transonic or supersonic flow conditions, therefore alters the thermophoretically-augmented *mass* deposition rate [3, 4]. Likewise, although transpiration has been extensively studied from the *heat* transfer (surface cooling) viewpoint, its effects on *mass* transport have previously only been estimated by invoking an inaccurate mass–heat transfer analogy. Indeed, in refs. [5, 6] we investigated how thermophoresis offsets the effect of real wall blowing, recovering earlier results [7] in the limiting cases of no thermal diffusion and constant properties.

A design engineer must deal with many nonlinear complications simultaneously. Therefore, a tractable conceptual approach to mass transfer predictions in the presence of thermophoresis, viscous dissipation, transpiration cooling and variable property effects has recently been suggested [1]. The present work, which implements this approach, develops rational, and general, yet simple correlations suitable for engineering predictions and optimizations. Pending the availability of reliable experimental data, the accuracy and the applicability limits of the correlations suggested here will be demonstrated using our exact numerical predictions for *laminar* or *turbulent* boundary layers.*

2. PROBLEM DEFINITION

We consider (cf. Fig. 1) a cooled object (e.g. heat exchanger tube or turbine blade), immersed in a hot fluid (e.g. post-combustion air) stream containing *trace* ($\omega_e \ll 1$) amounts of suspended vapors and/or small particles. Such an object will, of course, capture some of the vapors and small particles via the action of concentration (e.g. Brownian) diffusion in the immediate vicinity of the surface. However, *thermophoresis* (i.e. particle drift down the temperature gradient) is expected to dramatically increase the resulting deposition rate (by more than 1000-fold on a solid object at $T_w/T_e = 0.8$ and $p = 1$ atm. in a mainstream containing 1 μm diameter particles ($Sc \simeq 10^6$) [2]. In the case of transpiration-cooled gas turbine blades (Fig. 1), where a small portion of the compressor air is blown into the BL of hot combustion products, the surface temperature is dependent both upon the local flow rate and reservoir temperature of the injected gas. Therefore, while blowing *reduces* the rate at which particles can Brownian-diffuse to the wall [7], it also reduces the wall temperature, *increasing* the importance of *thermophoresis* (neglected in ref. [7], but included in the correlations below).

When the mainstream velocity is high enough, viscous dissipation effects alter the temperature profile within the thermal BL, causing an associated change in the rate of particle thermophoresis toward the surface.

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* To some it may seem premature to describe the present deposition rate correlations as 'engineering correlations', since in Section 3.4 we correlate more comprehensive BL *predictions*, rather than laboratory or field experimental data. Our view is that since the correlations and concepts in this paper can already be used to simplify *engineering predictions*, and, in the future, should simplify the reporting of *experimental deposition rate data* (e.g. the performance of particle capture equipment and aerosol instruments), the term engineering correlations *is* fully justified. Moreover, the most impressive prototypical correlations in the engineering literature (e.g. the dependence of the heat transfer Nusselt number on the Rayleigh group) have been suggested by *rational theories*—they are certainly not 'self-evident' from examining even large quantities of raw experimental data (e.g. natural convection heat transfer to/from selected Newtonian fluids).

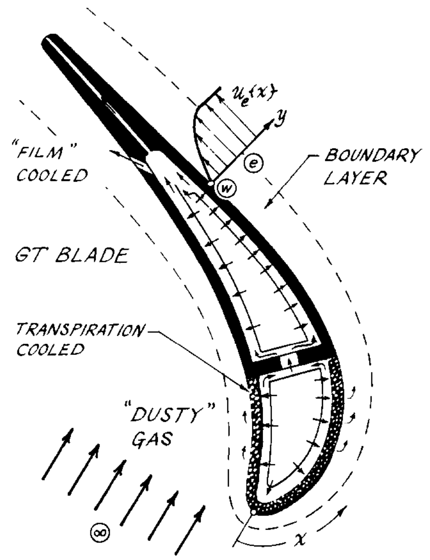

FIG. 1. Configuration and station nomenclature; example of transpiration-cooled and gas-film-cooled turbine blade immersed in a high velocity, hot 'dusty' gas stream.

It is interesting to note that whereas the phenomena of viscous dissipation and particle thermophoresis are each sufficient to 'break' the simplest form of the analogy between convective-diffusion *energy* and *mass* transfer, (1) the primary effect of viscous dissipation on *heat* transfer can be incorporated using an unaltered heat transfer *coefficient*, St_h together with a generalized 'driving force' (e.g. $T_{aw}-T_w$), whereas (2) the primary effect of particle thermophoresis (itself influenced by viscous dissipation) on the corresponding *mass transfer coefficient*, is to necessitate an St_m-correction factor [8] calculated and correlated below.

Our numerical calculations were done for (a) 'self-similar' laminar BLs where simple power-law temperature dependencies are assumed for the host gas (air) thermodynamic and particle transport properties, and (b) 'law-of-the-wall' *turbulent* BLs, with constant properties.

For non-zero wall blowing rates, the normal velocity at the wall, v_w, is considered specified, which, for a given coolant condition, also determines the wall temperature, T_w. The mainstream may have a *distribution* of particle size classes, but we treat only cases for which there is negligible chemical reaction, nucleation, condensation or coagulation *within* the two-dimensional BL. The wall is considered aerodynamically smooth and cool enough, or 'sticky' enough, so that $\omega_w \ll \omega_e$ (assumed constant).

3. BASIS OF THE CORRELATION

In mass transfer calculations neglecting thermophoresis the mass flux to the wall is given by the familiar expression

$$j''_{w,0} = -\rho_w D_{p,w}\left(\frac{\partial \omega}{\partial y}\right)_{w,0} \equiv \rho_e u_e St_0 \cdot (\omega_e - \omega_w), \quad (1)$$

where, in what follows, we will neglect ω_w compared to ω_e. In the case of transpiration cooling* or viscous dissipation, since $(\partial\omega/\partial y)_{w,0}$ is changed, the corresponding Stanton numbers are used. The calculation of these Stanton numbers in the absence of thermal diffusion has been thoroughly reviewed in the heat and mass transfer literature [9, 10].

In the presence of thermophoresis the diffusional flux to the wall is [1]

$$j''_w = -\rho_w D_{p,w}\left[\left(\frac{\partial \omega}{\partial y}\right)_w + \frac{\alpha_{T,w}\omega_w}{T_w}\cdot\left(\frac{\partial T}{\partial y}\right)_w\right], \quad (2a)$$

or, if $\omega_w \ll \omega_e$

$$j_w = -\rho_w D_{p,w}\left(\frac{\partial \omega}{\partial y}\right)_w = \rho_e u_e \, St \, \omega_e, \quad (2b)$$

since there is no explicit contribution from the second (thermophoretic drift) term in the brackets when $\omega_w \simeq 0$. However, even in this limit the presence of thermal diffusion affects the concentration profile [1], thereby altering $(\partial\omega/\partial y)_w$ and the corresponding Stanton number, St. Therefore, our correlations focus on the *ratio*, St/St_0.

3.1. *Correlating the effect of thermophoresis in the absence of transpiration cooling and viscous dissipation*

Thermophoresis introduces two types of new terms into the well-known mass transfer BL equations [1]: (1) a suction-like convective term due to the thermophoretic velocity, v_T, given by

$$v_T \equiv -D_p \cdot \alpha_{T,p} \cdot (\partial \ln T/\partial y), \quad (3)$$

and (2) a first-order reaction-like ('sink') term with an effective rate constant, k_{eff}, defined by

$$k_{eff} \equiv \frac{1}{\rho}\cdot\frac{\partial}{\partial y}\left(\rho D_p \alpha_{T,p}\cdot\frac{\partial \ln T}{\partial y}\right). \quad (4)$$

For transferred species heavier than the carrier gas ($Sc > 1$), mass transfer normal to the surface takes place throughout the *thermal* BL, within which the Brownian diffusion (*mass* transfer) BL is well imbedded. This is because thermophoresis is due to *temperature* gradients. In fact, for small particles ($Sc \gg 1$), the flux to the wall may be approximated by the thermophoretic flux 'feeding' the Brownian sublayer [3, 11, 12]; i.e.

$$j''_w \cong \lim_{y \to \delta_m} (\rho \omega v_T) = \rho_m \omega_m v_{T,m}. \quad (5)$$

The fact that the mass fraction of the transported species at the outer edge of the Brownian diffusion BL, ω_m, is different than the mainstream value, ω_e, can be attributed to the above-mentioned 'pseudo-sink' term operating *outside* the Brownian diffusion sublayer. However, even for vapor transport [$Sc = O(1)$], the corresponding 'sink' effect *inside* the mass transfer BL is negligible, as confirmed by our exact numerical calculations, and in ref. [13].

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* We assume here that only *clean* (dust-free) carrier gas (air) is blown through the walls.

In view of the above, we suggest that these 'suction' and 'sink' phenomena be treated separately, using two functions, fct_1 and fct_2, defined via

$$St/St_0 = fct_1\{\text{suction}\}\cdot fct_2\{\text{sink}\}$$
$$\approx fct_1\{\text{suction}\}\cdot fct_2\{\text{outer-sink}\}. \quad (6)$$

Here the 'outer-sink' function, fct_2, defined by

$$fct_2\{\text{outer-sink}\} \equiv \omega_{\mathrm{m}}/\omega_{\mathrm{e}}, \quad (7)$$

satisfies the limiting condition: $fct_2 \to 1$ for $Sc \to O(1)$.

As is discussed for analogous, previously studied heat and momentum transfer phenomena [10], we expect the 'thermophoretic suction' effect to depend on a dimensionless Peclet number based on the thermophoretic suction velocity and unmodified mass transfer BL thickness; i.e.

$$B_{\mathrm{T}} \equiv v_{\mathrm{T}}\delta_{\mathrm{m}}/D_{\mathrm{p}} \simeq -(\alpha_{\mathrm{T,p}}\ Le)_{\mathrm{w}}\cdot\frac{St_{\mathrm{h}}}{St_0}\cdot\frac{T_{\mathrm{aw}}-T_{\mathrm{w}}}{T_{\mathrm{w}}}, \quad (8a)$$

which, for laminar BLs, can be conveniently re-expressed as

$$B_{\mathrm{T}} \simeq -\alpha_{\mathrm{T,p,w}}\cdot(Le_{\mathrm{w}})^{1/3}\cdot\frac{T_{\mathrm{aw}}-T_{\mathrm{w}}}{T_{\mathrm{w}}}. \quad (8b)^*$$

An estimate to the functional form of fct_1 is provided by Couette-flow theory [1, 14]; i.e.

$$fct_1\{B_{\mathrm{T}}\} = \frac{-B_{\mathrm{T}}}{1-\exp(B_{\mathrm{T}})}, \quad (9)$$

which, in the limiting case of 'large' particles ($Sc \gg 1$ and $-B_{\mathrm{T}} \gg 1$), is also consistent with equation (5). This expression will be found to be more than adequate, when tested against our numerical calculations (see also Section 3.2).

Similarly, we expect the 'outer-sink' effect to depend on a dimensionless Damköhler-like group of the form

$$Da \equiv \frac{\bar{k}_{\mathrm{eff}}\delta_{\mathrm{h}}^2}{\alpha} \simeq (\alpha_{\mathrm{T,p}}\ Le)_{\mathrm{e}}\cdot\frac{T_{\mathrm{aw}}-T_{\mathrm{m}}}{T_{\mathrm{m}}}. \quad (10)\dagger$$

Note that here we use the more relevant *thermal* BL thickness and the mixture *heat* diffusivity. Since Brownian diffusion is not operating in the 'outer region', the BL equations in this region reduce to first-order differential equations. Therefore, an exponential function is proposed as a rough estimate to the form of fct_2, which plays the role of a reactor 'conversion' in the outer thermal BL, or (in the deposition rate expression) a 'reaction factor' (cf., e.g. ref. [15]). Thus, we investigated

$$fct_2\{Da\} = \exp(-Da) \quad (11)$$

—a choice which worked quite satisfactorily when checked against our numerical calculations, as well as those of refs. [3, 12]. Hence we propose

$$St/St_0 = fct_1\{B_{\mathrm{T}}\}\cdot fct_2\{Da\},$$

where fct_1, fct_2, B_{T} and Da are defined in equations (9), (11), (8) and (10), respectively.

3.2. *Correlating the effect of real suction and blowing in the absence of thermophoresis*

Blowing (or suction) effects on BLs have been extensively studied owing to heat transfer consequences, BL stability, separation, etc. [16]. To study its effects on small particle *mass* transfer in the absence of thermophoresis it is convenient to define the 'real' blowing (suction) parameter as

$$B_{\mathrm{m}} \equiv \frac{(\rho v)_{\mathrm{w}}}{(\rho u)_{\mathrm{e}}\ St_0}. \quad (13)$$

Of course, the sign of the normal velocity at the wall depends on whether there is blowing (positive) or sucking (negative) through the walls. As is well known, the functional form given in equation (9) successfully predicts the enhancement (reduction) of mass transfer rates by suction (blowing); i.e.

$$\frac{St_{0,B}}{St_0} \cong \frac{-B_{\mathrm{m}}}{1-\exp(B_{\mathrm{m}})} \quad \text{for } B_{\mathrm{m}} \leqslant 5. \quad (14)$$

For example, the experimental results given in ref. [17] on the reduction in heat transfer rates and skin friction by wall blowing (as well as our numerical results obtained as corollaries to the mass transfer calculations) also correlate well with equation (14), provided the relevant 'blowing parameters' are as defined in ref. [17] [similar to equation (13)]. For the diffusive flux of small particles to a wall under the simultaneous action of a constant force (i.e. gravity) in a stationary gas, one again obtains the same expression [18].

3.3. *Correlating the effect of viscous dissipation on thermophoresis in the absence of transpiration cooling*

Viscous dissipation modifies the BL temperature profile, and hence particle thermophoresis. This situation can be analyzed via two separate contributions (discussed in more detail in ref. [4]); i.e. (1) effects on 'thermophoretic suction' and the 'thermophoretic sink', which can be taken care of by using the more pertinent *adiabatic wall* temperature, T_{aw}, in place of the 'local mainstream' temperature, T_{e} [see equations (8) and (10)], and (2) additional effects on the 'thermophoretic sink'. The statement that for vapor deposition the 'thermophoretic sink' is negligible compared to the 'suction' effect remains valid in the presence of viscous dissipation up to Sc-values near 25; particle diameters of, say, 1.5×10^{-3} μm. For larger particles, however, we found an *additional* viscous

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* T_{aw} is the *adiabatic wall* temperature, given by $T_{\mathrm{aw}} = T_{\mathrm{e}}\cdot\{1+[(\gamma_{\mathrm{e}}-1)/2]\cdot r\cdot Ma_{\mathrm{e}}^2\}$, where r is the recovery factor, well-approximated by $r \approx (Pr)^{1/2}$ for laminar BLs.

† Here T_{m} is the temperature at the outer edge of the Brownian diffusion sublayer, as modified by thermophoresis, viz.: $T_{\mathrm{m}} = T_{\mathrm{w}} + Le^{s}\cdot(T_{\mathrm{e}}-T_{\mathrm{w}})$. The power, s, of the Lewis number is a measure of how the thickness of the Brownian diffusion sublayer changes with particle size (i.e. Sc). In the absence of thermophoresis $s = 1/3$; however, if thermophoresis is 'strong', say $B_{\mathrm{T}} \leqslant -5$, then $s = 1$. For $-5 \leqslant B_{\mathrm{T}} \leqslant 0$, a simple linear variation of s between 1/3 and 1 should suffice.

dissipation effect on the 'thermophoretic outer sink', adequately taken care of by a modifying post-factor, so that

$$fct_2\{Da,\ Ma_e\} \cong \left\{\exp\left[-(\alpha_{T,p}\ Le)_e \frac{T_{aw}-T_m}{T_m}\right]\right\} \cdot \left(\frac{T_{aw}}{T_e}\right)^{-5(\alpha_{T,p} Le)_e \cdot (T_w/T_e)} . \quad (15)$$

3.4. *Correlating the combined effect of viscous dissipation and transpiration cooling on thermophoresis*

We do not expect the introduction of transpiration cooling to appreciably modify the 'thermophoretic sink' term. Since the individual 'thermophoretic suction' and 'real suction (blowing)' effects have already been successfully expressed using the same functional form, we examine the postulate [11] that an algebraic combination of both 'blowing' parameters can be used in this single functional form. Thus, we here propose and examine the success of the comprehensive correlation

$$St/St_0 = fct_1\{B\} \cdot fct_2\{Da,\ Ma_e\}, \qquad (16)$$

where $B \equiv B_m + B_T$ and fct_1 is given by equations (9) and (14).

4. ACCURACY AND APPLICABILITY LIMITS OF THE CORRELATIONS

In developing the above-mentioned mass transfer rate correlations our aim is, of course, to cover the conditions of greatest engineering interest. Our numerical computations showed that St/St_0 is insensitive to (a) particle properties (tested for Na_2SO_4, SiO_2 and C), (b) Euler number [tested for $Eu = 1$ (forward stagnation point), $Eu = 0$ (flat plate), etc.], and (c) local mainstream temperature, T_e. Although the correlations have no explicit dependence on pressure, by virtue of the transition from the particle free-molecular regime [including *vapor* molecules with $Sc = O(1)$] to the continuum (Stokes–Einstein) regime ($Kn \ll 1$), the augmentation due to thermophoresis indeed depends on pressure, as indicated by our calculations at $p = 1$ and 20 atm. [2]. This effect is *implicitly* accounted for through the proper calculation of particle transport properties like D_p and $\alpha_{T,p}$[19]. In this connection, one important diffusivity *ratio* in our correlations is the product $\alpha_{T,p}$ *Le*, proportional to the particle 'thermophoretic diffusivity', $\alpha_{T,p}D_p$. Although kinetic theory calculations reveal that $\alpha_{T,p}$ *Le* is smaller than unity [21], the correlations are formally applicable in the range $0 \leqslant \alpha_{T,p}\ Le \leqslant 1.5$.

For a surface (e.g. heat exchanger tube) with a solid (impermeable) wall cooled by some internal mechanism, our correlations can be safely used in the temperature ratio range: $0.5 \leqslant T_w/T_e \leqslant 1$. However, for transpiration cooled surfaces we are, of course, limited to blowing rates such that the BL theory remains valid. For a typical ratio of coolant gas reservoir temperature to local mainstream temperature (say, 0.4) one can still use the correlations for *vapor* deposition at blowing rates as high as is necessary to bring the surface (wall) temperature down to $T_w/T_e \approx 0.8$. We note that, before the onset of inertial phenomena it is the largest particles ($Sc \gg 1$) which have the greatest difficulty reaching the surface by diffusion 'against' the blown coolant gas. In such cases a slight increase in blowing rate causes sharp reductions[*] in the deposition rate. We suggest, therefore, that this correlation *not* be used for values of $B \equiv B_m + B_T$[†] greater than about 5, corresponding to a reduction in deposition rate to about 1% of its original (solid wall) value.

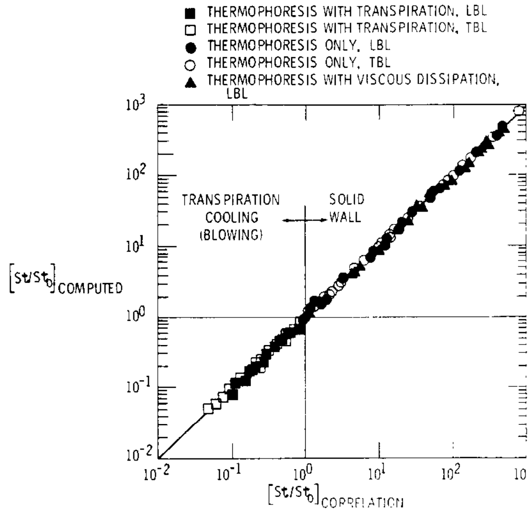

FIG. 2. Accuracy of BL mass transfer correlations in the presence of thermophoresis, viscous dissipation and transpiration cooling.

Provided conditions are such as to preclude appreciable particle inertia effects [11, 20, 21], under typical gas turbine environmental conditions, the correlations reported here can be used for particle diameters ranging from 0.4 nm [vapor molecules with $Sc = O(1)$] up to about 1 μm [i.e. $Sc = O(10^6)$], and viscous dissipation corresponding to Mach numbers between $0 \leqslant Ma_e \leqslant 2$ (current gas turbine blades operate at high subsonic Mach numbers).

Figure 2 displays the success of the correlations when tested against rigorous numerical BL calculations. For 'self-similar' *laminar* BLs with variable properties the average error is about 7%, with the maximum possible error being about 20%. For 'law-of-the-wall' *turbulent* BLs with constant properties (and C_f-equivalent pipe flow Reynolds numbers in the range: $0.9 \times 10^4 \leqslant Re \leqslant 3 \times 10^7$) even better results (average error of 2% and a maximum error of 10%) are

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[*] Much faster than our suggested correlation in equation (14) [5, 6].

[†] Note that B_m and B_T have opposite signs for 'real blowing' (positive) and 'thermophoretic suction' (negative).

obtained in the entire parameter range mentioned above. In all cases the accuracy improves as $Ma_e \to 0$, as $v_w \to 0$ ($T_w/T_e \to 1$) and as $T_w/T_e \to 1$ (for solid walls).

5. CONCLUSIONS AND RECOMMENDATIONS

To enable accurate particle deposition rate predictions in the presence of thermophoresis, and variable properties, possibly combined with wall transpiration cooling and/or viscous dissipation, without having to run a time-consuming computer program for each specific engineering design problem, comprehensive and rational, yet simple correlations have been developed. According to our approach the mass transfer coefficient ratio, St/St_0, is obtained from the product of two explicit functions [defined by equations (9), (15) and (16)], dependent only on accessible parameters defined by equations (8a), (8b) and (10). The present correlations (which are insensitive to particle properties, Eu and T_e, and only implicitly dependent on system pressure) gave an average error of 7% (with a maximum individual error of 20%) when tested against rigorous laminar BL calculations over the parameter ranges: $0 \leqslant Ma_e \leqslant 2$, $0.5 \leqslant T_w/T_e \leqslant 1$, $0 \leqslant \alpha_{T,p}\, Le \leqslant 1.5$ and $B \leqslant 5$; and an average error of 2% (maximum error of 10%) when tested against law-of-the-wall *turbulent* BL calculations over the parameter range: $0.5 \leqslant T_w/T_e \leqslant 1$, $0 \leqslant \alpha_{T,p}\, Le \leqslant 1.5$, $0.9 \times 10^4 \leqslant Re$ (pipe flow equivalent) $\leqslant 3 \times 10^7$ and $B \leqslant 5$ for all particle diameters ranging from vapor molecules (*ca.* 0.4 nm) up to about 1 μm.

The *local* applicability of convective-diffusion mass transfer correlations based on 'self-similar' BL solutions is currently being examined by testing its predictions against those obtained by incorporating particle thermophoresis into the two-dimensional BL code STAN 5 [22] for numerically solving prototypical *developing* BL situations (allowing for transition to turbulence, transpiration cooling and/or viscous dissipation [23]. However, it already appears that the correlation scheme suggested in refs. [1, 11, 20] and implemented here is remarkably general, successful and simple—more so, e.g. than the implementation of ref. [13] (which was limited to vapor transport to solid walls). Taken together with recent progress in the correlation of particle mass transfer mechanisms dominated by particle *inertia* [24], and a better understanding of the interaction between diffusional and inertial mechanisms [20], we believe the present ideas and results will facilitate future engineering predictions of particle deposition/removal rates in non-isothermal forced convection systems over the entire spectrum of mainstream particle sizes.

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FORMULATION DES VITESSES DE DEPOT DIFFUSIONEL DE PETITES PARTICULES MODIFIE PAR THERMOPHORESE, EN CONVECTION FORCEE AVEC DES PROPRIETES VARIABLES, UN REFROIDISSEMENT PAR TRANSPIRATION ET/OU UNE DISSIPATION VISQUEUSE

Résumé—Des formules sont données pour les calculs de bureau d'étude concernant les vitesses de dépôt diffusionnel de particules en présence de thermophorèse, de refroidissement par transpiration, de dissipation visqueuse et/ou de propriétés variables du fluide. On trouve que l'effet de ces phénomènes sur le nombre de Stanton de transfert massique peut être exprimé simplement par :

$$\frac{St_{\mathrm{m}}}{St_{\mathrm{m,0}}} \cong fct_1\left\{\begin{array}{c}\text{réel et/ou `soufflage'}\\ \text{(succion) thermophorétique}\end{array}\right\}\cdot fct_2\left\{\begin{array}{c}\text{`puits}\\ \text{thermophorétiques'}\end{array}\right\},$$

où la première fonction prend en compte le transfert de particules par le processus normal par convection sur la surface, et la seconde fonction rend compte de la variation spatiale de la vitesse de poussée thermophorétique, influencée par la dissipation visqueuse (principalement hors de la sous-couche de diffusion brownienne). Des formules de cette forme particulière, testées par des calculs numériques rigoureux de la couche limite (CL), donnent des erreurs *moyennes* de 7% seulement pour tous les cas de CL laminaires ($0 \leqslant Ma_{\mathrm{e}} \leqslant 2$; $0,5 \leqslant T_{\mathrm{w}}/T_{\mathrm{e}} \leqslant 1$; $0 \leqslant \alpha_{\mathrm{T,p}} Le \leqslant 1,5$; $B \leqslant 5$), et 2% pour tous les cas de CL turbulentes ($0,5 \leqslant T_{\mathrm{w}}/T_{\mathrm{e}} \leqslant 1$; $0 \leqslant \alpha_{\mathrm{T,p}} Le \leqslant 1,5$; $0,9 \times 10^4 \leqslant Re \leqslant 3 \times 10^7$; $B \leqslant 5$).

BEZIEHUNGEN FÜR DIE ABSCHEIDUNGSRATEN BEI DER THERMOPHORETISCHEN DIFFUSION KLEINER TEILCHEN IN ERZWUNGENEN KONVEKTIONSSTRÖMUNGEN MIT VARIABLEN STOFFWERTEN, VERDUNSTUNGSKÜHLUNG UND/ODER VISKOSER DISSIPATION

Zusammenfassung—Es werden vernünftige Beziehungen entwickelt und vorgestellt, um ingenieurmäßige Voraussagen über die Abscheidungsraten bei der Diffusion kleiner Teilchen bei vorhandener Thermophorese, Verdunstungskühlung, viskoser Dissipation und/oder variablen Fluideigenschaften machen zu können. Der Einfluß dieser Eigenschaften auf die Stantonzahl beim konvektiven Stofftransport läßt sich auf einfache Weise in Produktform darstellen:

$$\frac{St_{\mathrm{m}}}{St_{\mathrm{m,0}}} \cong fct_1\left\{\begin{array}{c}\text{reales und/oder thermophoretisches}\\ \text{`Ausblasen (Absaugen)'}\end{array}\right\}\cdot fct_2\left\{\begin{array}{c}\text{`thermophoretische}\\ \text{Senke'}\end{array}\right\}.$$

Dabei stellt die erste Funktion den Teilchentransport durch Konvektion senkrecht zur Abscheidungsoberfläche dar. Die zweite Funktion berücksichtigt die räumliche Verschiedenheit der thermophoretischen Driftgeschwindigkeit, wie sie durch viskose Dissipation (hauptsächlich außerhalb der Brown'schen Diffusionsunterschicht) beeinflußt wird. Die Beziehungen dieser besonderen Form wurden mit strengen numerischen Grenzschichtberechnungen verglichen. Für alle Fälle mit laminarer Grenzschicht ($0 \leqslant Ma_{\mathrm{e}} \leqslant 2$; $0,5 \leqslant T_{\mathrm{w}}/T_{\mathrm{e}} \leqslant 1$; $0 \leqslant \alpha_{\mathrm{T,p}}\, Le \leqslant 1,5$; $B \leqslant 5$) ergaben sich mittlere Fehler von nur 7%, für alle Fälle mit turbulenter Grenzschicht ($0,5 \leqslant T_{\mathrm{w}}/T_{\mathrm{e}} \leqslant 1$; $0 \leqslant \alpha_{\mathrm{T,p}}\, Le \leqslant 1,5$; $0,9 \times 10^4 \leqslant Re \leqslant 3 \times 10^7$; $B \leqslant 5$) lagen die mittleren Fehler bei 2%.

ЗАВИСИМОСТИ ДЛЯ СКОРОСТЕЙ ДИФФУЗИОННОГО ОСАЖДЕНИЯ МЕЛКИХ ЧАСТИЦ, ОБУСЛОВЛЕННОГО ТЕРМОФОРЕЗОМ, В ВЫНУЖДЕННЫХ КОНВЕКТИВНЫХ СИСТЕМАХ С ПЕРЕМЕННЫМИ СВОЙСТВАМИ ПРИ ИСПАРИТЕЛЬНОМ ОХЛАЖДЕНИИ И/ИЛИ ВЯЗКОЙ ДИССИПАЦИИ

Аннотация—Рассчитаны и представлены критериальные зависимости для инженерных расчетов скоростей диффузионного осаждения мелких частиц в случае термофореза, испарительного охлаждения, вязкой диссипации и/или переменных свойств жидкости. Высказано предположение и установлено, что влияние этих явлений на число Стэнтона для конвективно-диффузионного массопереноса можно выразить в виде

$$\frac{St_m}{St_{m,0}} \cong fct_1\left(\begin{matrix}\text{реальный и/или термофорезный}\\ \text{``вдув'' (отсос)}\end{matrix}\right)\cdot fct_2\left(\begin{matrix}\text{``термофорезный}\\ \text{сток'' (источник)}\end{matrix}\right),$$

где первая функция описывает перенос частиц конвективными процессами, нормальными к поверхности осаждения, а вторая—пространственную неоднородность индуцированной термофорезом скорости дрейфа при учете влияния вязкой диссипации (вне подслоя, создаваемого броуновской диффузией). Зависимости такого вида при сравнении с точными численными расчетами для пограничного слоя дают средние ошибки порядка 7% для всех случаев *ламинарных* пограничных слоев ($0 \leq Ma_e \leq 2$; $0{,}5 \leq T_w/T_e \leq 1$; $0 \leq \alpha_{T,p} Le \leq 1{,}5$; $B \leq 5$) и 2% для всех случаев *турбулентных* пограничных слоев ($0{,}5 \leq T_w/T_e \leq 1$; $0 \leq \alpha_{T,p} Le \leq 1{,}5$; $0{,}9 \times 10^4 \leq Re \leq 3 \times 10^7$; $B \leq 5$), соответственно.